2.1 Product and Quotient Rules: Form B Assessment

Name: _____ Time: 60 minutes No calculator 100 points

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Answer exactly the question asked. Show work on every question except Question 11. Angles are in radians. If a derivative does not exist, write DNE. If a function value does not exist, write *undefined*.

1. Find $f'(x)$ for each function. **(3 each, 24 pts)**
   You need not list x-values where $f'(x)$ does not exist.

   (a) $f(x) = \sqrt{x}\,\sin x$

   (b) $f(x) = (3x+1)(x^2-2)$

   (c) $f(x) = \dfrac{x+2}{x^2+5}$

   (d) $f(x) = \dfrac{6x^4-2x^2}{2x^3}$

   (e) $f(x) = \dfrac{2x^3-6x^2}{x^3-3x^2}$

   (f) $f(x) = 4\csc x + \sin x \csc x$

   (g) $f(x) = x^2 \cot x$

   (h) $f(x) = \sec\left(\frac{\pi}{3}\right)\cos(\pi)$

2. Let $f(x) = \dfrac{\cos x}{1+\sin x}$. Find $f'\left(\frac{\pi}{6}\right)$. **(5 pts)**

3. Evaluate each limit. **(3 each, 6 pts)**

   (a) $\displaystyle\lim_{h\to 0} \frac{\sec\left(\frac{\pi}{3}+h\right)-2}{h}$

   (b) $\displaystyle\lim_{x\to \pi/2} \frac{\cot x}{x-\frac{\pi}{2}}$

4. Let $f(x) = \dfrac{x+3}{x-1}$. Find an equation of the tangent line to the graph of f at $x = 3$. **(6 pts)**

5. Let $f(x) = \sqrt{x^3+1}$. **(5 pts)**
The line tangent to the graph of f at $x = 2$ passes through the point $(-1, -3)$. Find $f'(2)$.

6. Let $f(x) = 4x + \cot x$. **(6 pts)**
Find every x in the interval $(0, \pi)$ at which the graph of f has a horizontal tangent line.

7. Derive the rule for the derivative of $\cot x$. **(6 pts)**
Write $\cot x = \dfrac{\cos x}{\sin x}$ and use the quotient rule to show that the derivative of $\cot x$ is $-\csc^2 x$.
Quoting the rule for $\cot x$ earns no credit.

8. The table gives selected values of f, g, and their derivatives. **(4 each, 12 pts)**

Let $h(x) = \dfrac{g(x)}{f(x)}$, $k(x) = f(x)\,g(x)$, and $m(x) = \dfrac{g(x)}{f(1)}$.

x	$f(x)$	$g'(x)$	$f'(x)$	$g(x)$
-2	3	4	-1	2
1	2	2	1	-3
3	-1	-2	4	3

(a) Find $h'(1)$.

(b) Find $k'(3)$.

(c) Find $m'(-2)$.

9. Each part describes a different function. **(4 each, 12 pts)**

(a) Let $h(x) = f(x)\,g(x)$. It is known that $f(-1) = -2$, $g(-1) = 3$, $g'(-1) = 1$, and $h'(-1) = 7$. Find $f'(-1)$.

(b) Let $k(x) = \dfrac{x^2}{g(x)}$. It is known that $g(2) = 4$ and $k'(2) = -\frac{1}{4}$. Find $g'(2)$.

(c) Let $w(x) = \dfrac{f(x)}{x}$. It is known that $f'(2) = 5$ and $w'(2) = 1$. Find $f(2)$.

10. Figure B10 shows the graphs of f (solid) and g (dashed). **(4 each, 12 pts)**

Each graph is made of straight line segments that meet at grid points. Let $q(x) = \dfrac{g(x)}{f(x)}$, $p(x) = f(x)\,g(x)$, and $r(x) = \dfrac{f(x)}{x}$.

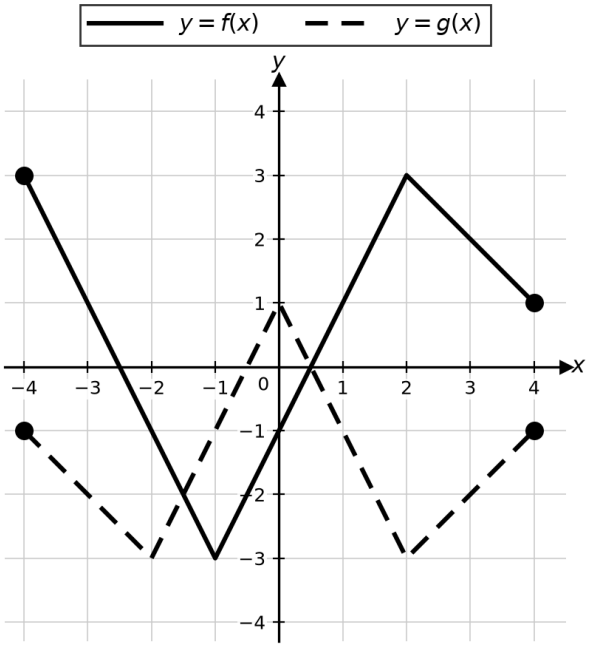


Figure B10

(a) Find $q'(-3)$.

(b) Find $p'(3)$.

(c) Find $r'(1)$.

11. Each graph below shows two curves on the same axes. **(6 pts)**

In exactly one of the graphs, one curve is the derivative of the other. Circle the Roman numeral of that graph.

